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Are all highly liquid securities within the same class?

S.M.D. Queirós[a]

Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud 150, 22290-180, Rio de Janeiro-RJ, Brazil

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Abstract. In this article we analyse the leading statistical properties of fluctuations of (log) 3-month US Treasury bill quotation in the secondary market, namely: probability density function, autocorrelation, absolute values autocorrelation, and absolute values persistency. We verify that this financial instrument, in spite of its high liquidity, shows very peculiar properties. Particularly, we verify that log-fluctuations belong to the Lévy class of stochastic variables.

PACS. 05.90.+m Other topics in statistical physics, thermodynamics, and nonlinear dynamical systems – 05.45.Tp Time series analysis – 89.65.Gh Economics; econophysics, financial markets, business and management

Financial markets have become a paradigmatic example of complexity and the focus of plenty of work within physics. Specifically, several techniques, mainly related to statistical physics (e.g., stochastic dynamics, theory of critical phenomena or nonlinear systems), have been applied either to reproduce or simply verify several properties, e.g., the probability density functional form (PDF), or the autocorrelation function (ACF) of financial observables [1–3]. The systematic (asymptotic) power-law behaviour found for quantities such as price/index fluctuations, or traded volume has been pointed out to be at the helm of the multifractal character of financial time series [4], a feature that is also regular in out-of-equilibrium systems [5]. On the account of the background on this type of phenomena, in which scale invariance also rules, it has come out the endeavour to identify universality classes for financial markets defined by the exponents that characterise their main statistical properties. Explicitly, these classes indicate the existence of a common behaviour for systems within the same class apart their microscopic or specific details [6]. On this way, it has been suggested [6] that financial products like securities with a very high level of liquidity (high trading activity) might present similar characteristics. As an example, it has been shown that, despite of the fact that in their essence stocks and commodities are completely different financial instruments (securities), their (daily) price fluctuations behave on a very similar way, i.e., they can be enclosed in the same class [7].

Within securities are also public debt bonds like United States (US) Treasury bills [8,9]. The US Treasury bills (T-bills) are marketable bonds issued by the US federal government and represent one of the debt financing instruments used by the Treasury Department [10]. T-bills are classified as *zero-coupon* bonds that are sold in the primary market at a discount of the face value in order to present a positive yield to maturity which can be 28 (1 month), 91 (3 months), or 182 (6 months) days. In regard of this, they are considered to be the most risk-free investment in the USA. This makes of T-bills an important and heavily traded (i.e., highly liquid) financial instrument in the secondary market where they are quoted on an annual percentage yield to maturity.

In the sequel of this article we study some of the main statistical features of the 3-month US T-bills traded on the secondary market. Our time series, $\{Q_t\}$, which is named $DTB3$ by the Federal Reserve, is composed by 3-month US T-bill daily prices and runs from the 4th January 1954 up to the 26th February 2007 in a total of 13866 trading days [11]. Our choice for a maturity of 3 months is justified by the fact that it is the most used interest rate maturity in derivative financial products like call-put options. To compare the statistical properties of $DTB3$ daily log-value fluctuations, $\tilde{r}_t = \ln Q_t - \ln Q_{t-1}$, we use the daily log-index fluctuations, $\tilde{r}'_t = \ln S_t - \ln S_{t-1}$, of $SP500$ time series, $\{S_t\}$, which runs the same time interval as $DTB3$. Both fluctuation time series, $\{\tilde{r}_t\}$ and $\{\tilde{r}'_t\}$ have been subtracted of respective averages, $\left\langle \tilde{r}_t^{(\prime)} \right\rangle$, and normalised by standard deviation $\sigma_{\tilde{r}^{(\prime)}}$, i.e., $r_t^{(\prime)} = \left[\tilde{r}_t^{(\prime)} - \left\langle \tilde{r}_t^{(\prime)} \right\rangle\right] / \sigma_{\tilde{r}^{(\prime)}}$. (from here on the prime stands for $SP500$ quantities, and

[a] *Present address:* Unilever R&D Port Sunlight, Quarry Road East, Wirral CH63 3JW, UK
e-mail: `sdqueiro@cbpf.br`

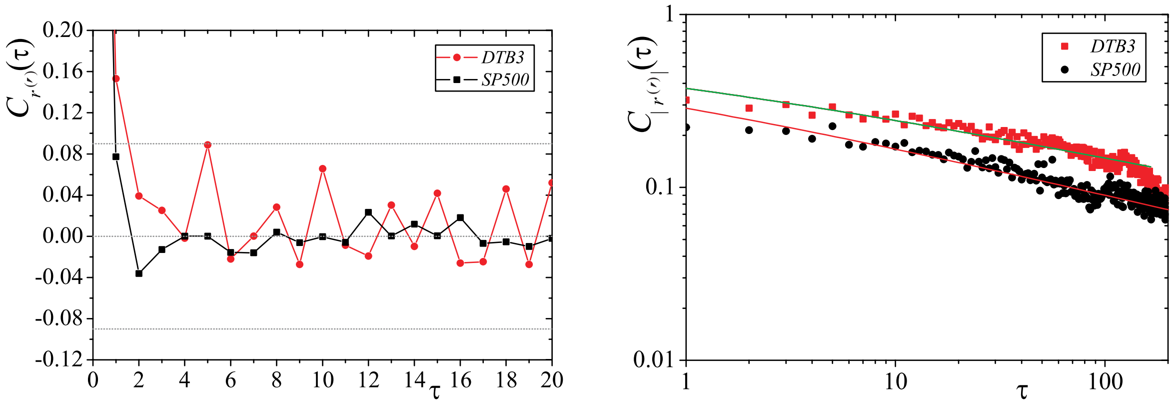

Fig. 1. (Colour online) Autocorrelation function equation 1, $C(\tau)$ vs. τ of $r_t^{(\prime)}$ (left panel) and $\left|r_t^{(\prime)}\right|$ in a log-log scale (right panel). It is visible that r_t is correlated for immediate correlations and presents measurable correlations every multiple of 5 days lag. The autocorrelation of $\left|r_t^{(\prime)}\right|$ can be described by equation (2) with $q_c = 4.7 \pm 0.1$, $\mathcal{T} = 0.45 \pm 0.05$ ($\chi^2/n = 2 \times 10^{-4}$, $R^2 = 0.9$) for $DTB3$, and $q_c' = 4.3 \pm 0.1$, $\mathcal{T}' = 0.77 \pm 0.05$ ($\chi^2/n = 10^{-4}$, $R^2 = 0.9$) for $SP500$. The dashed line in left panel represents three times the noise level bounds.

x is used in definitions to represent any observable upon analysis).

Moving ahead, we shall now analyse and compare primary and more usually studied statistical features. Commencing with the analysis of ACF,

$$C_x(\tau) = \frac{\langle x_t\, x_{t+\tau}\rangle - \langle x_t\rangle^2}{\langle x_t^2\rangle - \langle x_t\rangle^2}, \qquad (1)$$

we have verified a noteworthy difference between $\{r_t\}$ and $\{r_t'\}$. Firstly, as depicted in Figure 1, $C_r(1)$ clearly exceeds three time noise level within which typical interday correlation values of $SP500$ and other indices as well [1] lay in. Additionally, correlation values greater than noise level have been measured at least for lag $\tau = 5, 10$ days. We attribute the origin of this feature to the fact that T-bills are weekly (5 trading days) sold at the primary market. Concerning the ACF of absolute values, we have not observed any relevant differences. Both curves are fairly described by (asymptotic power-law) $q_c^{(\prime)}$-exponential functions,

$$C_{\left|r^{(\prime)}\right|}(\tau) = \left[1 - \left(1 - q_c^{(\prime)}\right)\mathcal{T}^{(\prime)}\,\tau^2\right]^{\frac{1}{1-q_c^{(\prime)}}}, \qquad (2)$$

where $q_c^{(\prime)}$ gives the decaying exponent, and $\mathcal{T}^{(\prime)}$ characteristic parameter. The value $q_c = 4.7 \pm 0.1$ is not far from $q_c' = 4.3 \pm 0.1$, and both are in accordance with previous values obtained for $SP500$ [12] or $DJIA$ equities [13]. For $\mathcal{T}^{(\prime)}$ we have obtained $\mathcal{T} = 0.45 \pm 0.05$, and $\mathcal{T}' = 0.77 \pm 0.05$.

Stronger dissimilarity has appeared on the PDFs, which we have fitted for q-Gaussian distributions,

$$\mathcal{G}_q(x) = \mathcal{A}\left[1 - (1-q)\,\mathcal{B}\,x^2\right]^{\frac{1}{1-q}}, \qquad (q<3), \qquad (3)$$

where $\mathcal{A}$ is the normalisation, and $\mathcal{B}$ is related to the "width" of the distribution determined by its q-generalised second order moment, $\sigma_q^2 = \int x^2\ [P(x)]^q\ dx / \int [P(x)]^q\, dx$, in the form, $\mathcal{B} = \left[\sigma_q^2\,(3\,q-1)\right]^{-1}$ [14]. When $q < 5/3$, standard deviation is finite and the equality $\mathcal{B} = \left[\sigma^2\,(5-3\,q)\right]^{-1}$ is also valid. For $q < 3$, Distribution (3) emerges from optimising non-additive (Tsallis) entropy upon appropriate constraints [15]. In the limit $q \to 1$ the Gaussian distribution is obtained, $\mathcal{G}_1(x) \equiv \mathcal{G}(x)$. Regardless both of the two fluctuations are well described by equation (3), the values of q are qualitatively quite different. Namely, we have obtained the best fit for $q = 1.72 \pm 0.02$ for $DTB3$, and $q' = 1.49 \pm 0.01$ for $SP500$ (see Fig. 2)[1]. The latter is in accordance with prior analysis [1–3,12,16]. Such a disparity has clear implications on the attractor in probability space of each observable when we consider the addition of fluctuations defining variable $R_{N,t}^{(\prime)} \equiv \sum_{i=0}^{N-1} r_{t+i}^{(\prime)}$. Since the two signals are essentially uncorrelated, in the sense that ACF rapidly attains at noise level, standard central limit theorems do apply [17]. In other words, for $SP500$, by reason of its entropic index q is smaller than $\frac{5}{3}$, σ' is finite, hence the convolution of PDF log-$SP500$ fluctuations leads to the Gaussian distribution, $\mathcal{G}(R_N') = \frac{1}{\sqrt{2\,\pi\,N\,(\sigma')^2}} \exp\left[-\frac{R_N'}{2\,N\,(\sigma')^2}\right]$ (for $N \to \infty$, and since the daily time series has been normalised upon a finite series, $\sigma' \approx 1$). Conversely, the entropic index for $DTB3$ is greater than $\frac{5}{3}$, which makes σ actually incommensurable. Thus, according to the Lévy-Gnedenko central limit theorem [17], the attracting distribution (for $N \to \infty$) is an α-stable distribution,

$$\mathcal{L}_\alpha(R_N) = \frac{1}{2\,\pi}\int_{-\infty}^{+\infty} \exp\left[-i\,k\,R_N - a\,|k|^\alpha\right] dk, \qquad (4)$$

[1] We have also used the Hill estimator to evaluate tail exponents. Due to series length and error margins we cannot rely on the results obtained by this method, although considering error margins they accord with $q^{(\prime)}$ values.

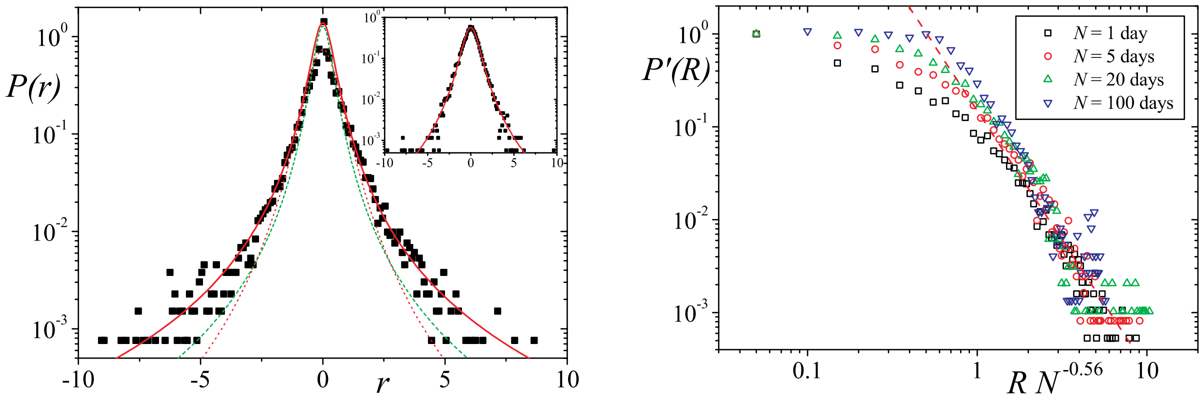

Fig. 2. (Colour online) Left panel: PDF, $P(r)$ vs. r in log-linear scale. Symbols are obtained from data and the full line represents the best numerical adjustment for equation (3), with $q = 1.72 \pm 0.02$ and $\mathcal{B} = 5.9 \pm 0.4$ ($R^2 = 0.96$ and $\chi^2/n = 3 \times 10^{-3}$). The dotted line is the best fit for $\mathcal{G}_q(r)$, but imposing $q = q' = 1.49$ as in $SP500$ case shown at the in-set ($\mathcal{B}' = 2.23 \pm 0.09$, $R^2 = 0.99$, and $\chi^2/n = 3 \times 10^{-4}$). The dashed line represents the best fit with $q = 1.666$ ($\mathcal{B} = 8.2 \pm 0.6$) (on the edge of finite variance). Right panel: $P'(R_N) = \frac{P(R_N)}{P(0)} N^{1/\alpha \cdot}$ vs. $R_N\, N^{-1/\alpha}$ for $N = 1, 5, 20, 100$ days in log-log scale. The asymptotic collapse of the curves, described by a tail exponent of $1 + \alpha = 2\frac{1}{q-1} = 2.77$ is visible.

with $\alpha = (3-q)/(q-1)$, which follows, for large N, the scaling law $\mathcal{L}_\alpha(R_N) = N^{-1/\alpha} \mathcal{L}_\alpha\left(\frac{R_N}{N^{1/\alpha}}\right)$, and $\mathcal{L}_\alpha(R_N) \sim R_N^{-\alpha-1}$. As it is visible in Figure 2, the PDFs of properly scaled R_N variables obtained from $r(t)$ signal asymptotically collapse exhibiting a tail described by $\alpha \approx 1.77$, as it happens for variables whose attractor is a α-stable distribution (see Ref. [18]). This constitutes, in our point of view, a substantial difference between 3-month T-bill daily fluctuations and other financial fluctuations, by the fact that it represents a drastic change of the attractor.

Within a macroscopic framework, the long-lasting form of the absolute price fluctuations ACF has been held responsible for the non-Gaussian behaviour of financial securities fluctuations [19,20]. To further analyse the persistency of absolute fluctuations, we have applied the *DFA* method to assess the Hurst exponent, H, of $\left|r_t^{(\prime)}\right|$ time series and shuffled $\left\{\left|r_t^{(\prime)}\right|\right\}$ (procedure presented in [21]). The results are exhibited in Figure 3, where N represents the length of the time series [21]. For $N > 40$ we have verified that $DTB3$ presents a strong persistent behaviour as $SP500$ does with $H = 0.90 \pm 0.02$ and $H' = 0.90 \pm 0.03$. For $N < 40$ we verify a crossover, but this time index and T-bill fall apart with $H = 0.50 \pm 0.02$ (like a Brownian motion) and a specious $H' = 0.27$ (antipersistency). It is known that the presence of spikes and locality on persistency might introduce spurious features on DFA analysis [22–24] of persistent signals leading to $H < 1/2$ values for small N. We attribute to this fact the emergence of $H \leq 1/2$ values.

Another property we have analysed are the correlations between fluctuations and absolute fluctuations [25],

$$L(\tau) = \frac{\left\langle r_t^{(\prime)} \left[r_{t+\tau}^{(\prime)}\right]^2 \right\rangle}{\left\langle \left[r_{t+\tau}^{(\prime)}\right]^2 \right\rangle^2}. \qquad (5)$$

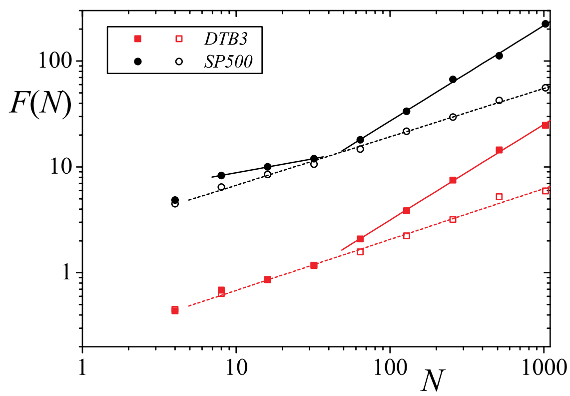

Fig. 3. (Colour online) Left panel: Root-mean-square deviation $F(N)$ vs. N for integrated absolute fluctuations time series of $DTB3$ (squares), and $SP500$ (circles). The full symbols are from the ordered series and empty symbols from shuffled signals. For large N we have measured a Hurst exponent of 0.90 ± 0.02 ($DTB3$), and 0.90 ± 0.03 ($SP500$). For small N, T-bills absolute fluctuations log-fluctuations presents a behaviour similar to white noise while $SP500$ exhibits a antipersistent behaviour.

It has been verified in several securities and financial indices that $L(\tau) = 0$ for $\tau < 0$, and $L(\tau) \sim -\exp[-\tau/\lambda]$ for $\tau \geq 0$. This behaviour, known has *leverage effect* [26], is intimately related to risk-aversion and negative skew of price fluctuations PDF. In defiance of the noisy $L(\tau)$ which has inhibited us to present a trusty quantitative description, it is plausible to affirm that $DTB3$ fluctuations also show time symmetry breaking, but in an *antisymmetrical fashion*, $L(-\tau) = -L(\tau)$, as it is understandable from Figure 4. For $\tau < 0$, there is a positive correlation between fluctuations and future absolute fluctuations, whereas for $\tau > 0$, there exists a negative

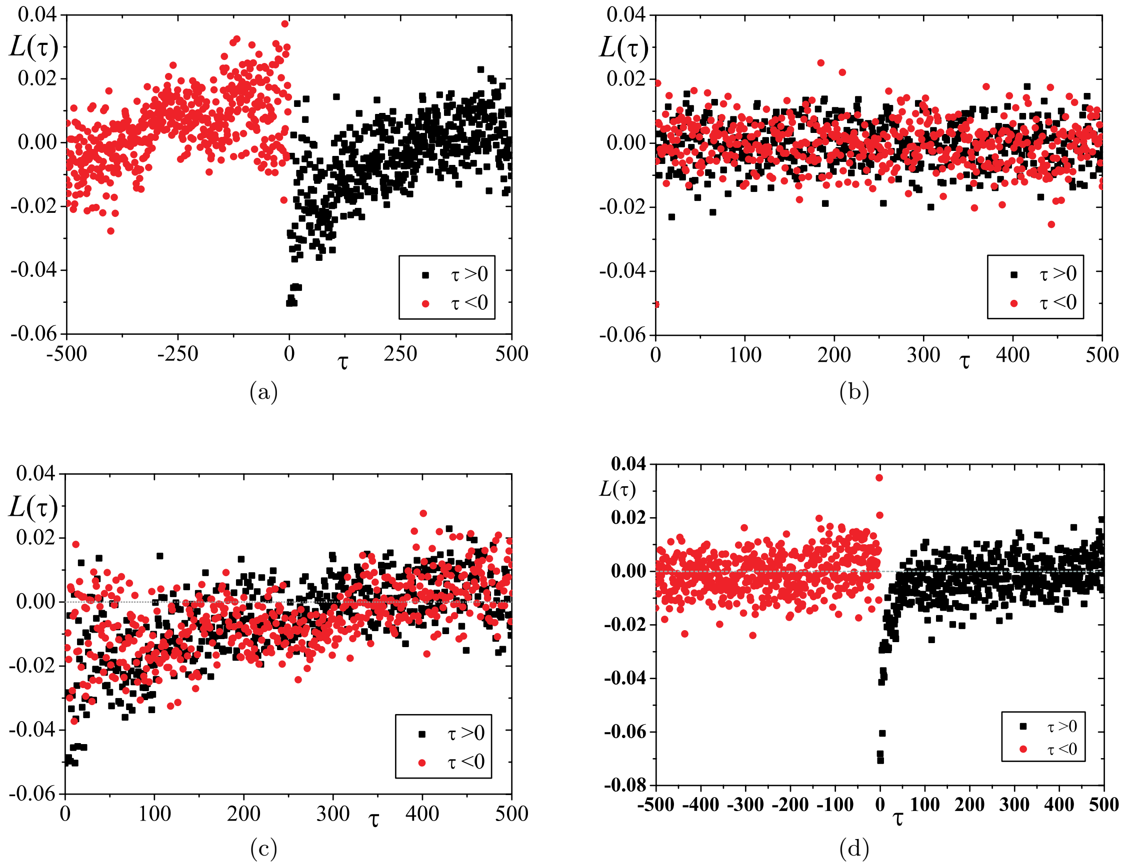

Fig. 4. (Colour online) $L(\tau)$ vs. τ of $DTB3$ (a), and shuffled $\{r_t\}$ (b). Comparing both panels and taking into account noise level (dashed lines) it is visible the existence of a functional form for $L(\tau)$. Panel (c): $L(\tau)$ for $\tau > 0$ and $-L(-\tau)$ for $\tau < 0$ vs. τ. At panel (d), $L(\tau)$ vs. τ of $SP500$ for mere illustration purposes.

correlation between fluctuations and future absolute fluctuations. This antisymmetric behaviour has clear implications on dynamical mimicking. As an example, the Heston approach to financial fluctuations [27,28], in which the noises of stochastic equations for the fluctuation and instantaneous variance are anti-correlated, must be modified in order embrace our empirical observations of T-bill log-fluctuations.

To summarise, in this article we have analysed a set of statistical properties of daily fluctuations of the 3-month T-bill trading value, a highly liquid security. Our results have shown important differences between this financial instrument and a paradigmatic example of financial securities statistical properties, the daily fluctuations of $SP500$ index which also presents similar properties to other debt bonds [1]. Specifically, we have verified that T-bill daily fluctuations PDF belong to the α-stable class of distributions, while other liquid securities that have been studied so far present the Gaussian distribution as the attractor in PDF space. This represents a fundamental justification for the well-known difficulties on the construction (namely specification) and implementation (namely identification and estimation) of generalised spot interest rate models [29], which are always built assuming a finite standard deviation, unlike Lévy-Gnedenko class of random variables. Moreover, we have unveiled that the fluctuations-fluctuations magnitude correlation function presents an antisymmetric form, i.e., a different behaviour than the "leverage effect" that has been verified in other securities.

Our results emphasise the idea that liquidity is not the only factor to take into account when we aim to define a behavioural class for financial securities [7,30]. Properties such as the nature of the financial instrument under trading are actually relevant for its dynamics and categorisation. We address to future work the development of dynamical scenarios capable of reproducing the statistical properties we have presented herein.

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